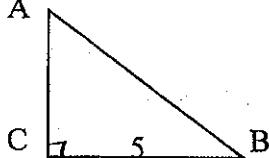


Precalculus Unit 3 Test Review: Unit Circle and Trig IdentitiesROUND TO THE NEAREST TENTH UNLESS OTHERWISE SPECIFIED.

Part 1. Find the information listed for each triangle.

a. A

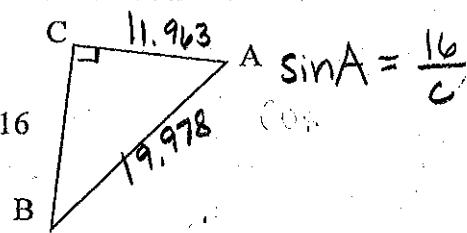


$$\sin 42^\circ 15' = \frac{5}{c}$$

$$\tan 42^\circ 15' = \frac{5}{b}$$

$\angle A = 42^\circ 15'$	$\sin A = \frac{5}{7.436} = .6723$
$\angle B = 47^\circ 45'$	$\cos A = \frac{5.504}{7.436} = .7402$
$\angle C = 90^\circ$	$\tan A = \frac{5}{5.504} = .9083$
$a = 5$	$\csc A = \frac{7.436}{5} = 1.4873$
$b = 5.504$	$\sec A = \frac{7.436}{5.504} = 1.3510$
$c = 7.436$	$\cot A = \frac{5.504}{7.436} = 1.1009$

b.



$$\sin A = \frac{16}{c}$$

$$\cos A = \frac{19.978}{c}$$

$\angle A = 53^\circ 12' 52''$	$\sin A = \frac{16}{19.978} = .8009$
$\angle B = 36^\circ 47' 8''$	$\cos A = \frac{19.978}{16} = .5988$
$\angle C = 90^\circ$	$\tan A = \frac{16}{19.978} = 1.3375$
$a = 16$	$\csc A = \frac{19.978}{16} = 1.2486$
$b = 19.978$	$\sec A = \frac{16}{19.978} = 1.6699$
$c = 24.000$	$\cot A = \frac{19.978}{16} = 1.2486$

Part 2. ABC Triangles: Find the missing values of the triangles, if possible. Give angles measures in DMS.

a. $\angle A = 72^\circ 15' 8''$ $a = 22.86$

$\angle B = 17^\circ 44' 52''$ $b = 7.316$

$\angle C = 90^\circ$ $c = 24$

$$\sin A = \frac{a}{c} \quad 22.86^2 + b^2 = 24^2$$

$$b^2 = 53.52$$

b. $\angle A =$ $a = 24$

$\angle B = 35^\circ 55' 10''$ $b = 10$

$\angle C =$ $c =$

$$\frac{\sin 35^\circ 55' 10''}{10} = \frac{\sin A}{24} \quad (\text{not a } \Delta)$$

NO
SOLUTION

c. $\angle A = 18^\circ 27' 1.173''$ $a = 8.385$

$\angle B = 122^\circ 11' 45''$ $b = 22.42$

$\angle C = 39^\circ 21' 13.8''$ $c = 16.8$

$$\frac{\sin 122^\circ 11' 45''}{22.42} = \frac{\sin C}{16.8} = \frac{\sin A}{a}$$

d. $\angle A = 23^\circ 3' 12''$ $a = 15$

$\angle B = 109^\circ 58' 42.4''$ $b = 36$

$\angle C = 46^\circ 58' 4.6''$ $c = 28$

**Don't find m<A (smallest angle) first!

$$36^2 = 15^2 + 28^2 - 2(15)(28)\cos B$$

$$28^2 = 15^2 + 36^2 - 2(15)(36)\cos C$$

Part 3. Use your calculator to determine the values of each. Round to the nearest thousandth.

a. $\sin(73^\circ) = .956$

b. $\cot(336^\circ) = -2.246$

c. $\csc(128^\circ) = 1.269$

d. $\tan\left(\frac{11\pi}{8}\right) = 2.414$

e. $\sec(163^\circ) = -1.046$

f. $\cos\left(\frac{\pi}{12}\right) = .966$

Part 4. Applications of Solving Triangles"

~~See work @ end of document.~~

- a. At a certain time of day, the angle of elevation of the sun is $49^{\circ}53'19''$. Find the length of the shadow of a tree that is 28 feet tall.
 $\boxed{23,588 \text{ ft}}$
- b. A plane is coming in to land at an airport. Its angle of descent is $5^{\circ}38'15''$. If the altitude of the plane is 30,000 ft., find the horizontal distance (distance along the ground) to the airport.
 $\boxed{303919.37 \text{ ft}}$
- c. A tower is 150 feet tall. Its shadow caused by the sun is 42 feet long. Find the angle of depression from the sun to the tip of the tower's shadow. Give your answer in DMS.
 $\boxed{74^{\circ}21'27.913''}$
- d. The angle of elevation of the top of a mountain peak as observed from Stanley on the level plain below is $53^{\circ}27'42''$. From the point at which Oscar is standing on the same level plain 400 feet closer to the mountain, the angle of elevation is $29^{\circ}15'56''$. Find the height of the peak.
 $\boxed{383,327 \text{ ft}}$ further from
- e. Two planes take off at the same time from the same airport. One is traveling 810 km per hour on a bearing of N $15^{\circ}10'8''$ W. The other is traveling as 925 km per hour at a bearing of S $82^{\circ}55'42''$ W. How far apart are the planes after 2 hours?
 $\boxed{2625.105 \text{ km}}$

Part 5: Degrees and Radians

Change the following degrees to radians.

a. 330°

$$\frac{11\pi}{6}$$

b. -125°

$$-\frac{25\pi}{36}$$

c. 160°

$$\frac{8\pi}{9}$$

Change the following radians to degrees.

a. $\frac{\pi}{6}$

$$30^{\circ}$$

b. $\frac{3\pi}{5}$

$$108^{\circ}$$

c. $\frac{11\pi}{6}$

$$330^{\circ}$$

Part 6. Co-terminal Angles – Find the positive and negative co-terminal angles for each.

a. $\frac{11\pi}{6}$

$$\frac{23\pi}{6}$$

$$-\frac{7\pi}{6}$$

b. $\pi =$

$$3\pi$$

$$-\pi$$

c. $\frac{\pi}{3} =$

$$\frac{2\pi}{3}$$

$$-\frac{5\pi}{3}$$

d. $-\frac{4\pi}{3}$

$$\frac{2\pi}{3}$$

$$-\frac{10\pi}{3}$$

e. $-\frac{5\pi}{3} =$

$$\frac{\pi}{3}$$

$$-\frac{11\pi}{3}$$

f. $160^{\circ} =$

$$520^{\circ}$$

$$-200^{\circ}$$

g. $270^{\circ} =$

$$630^{\circ}$$

$$-90^{\circ}$$

h. $25^{\circ} =$

$$385^{\circ}$$

$$-335^{\circ}$$

i. $-133^{\circ} =$

$$227^{\circ}$$

$$-493^{\circ}$$

Part 7: Please give: a. The reference angle in question; b. the quadrant for the angle in question c. The trig value

a. $\sin(120^{\circ}) = \frac{\sqrt{3}}{2}$ RA: 60° Quad: 2	b. $\sec(\frac{3\pi}{4}) = -\sqrt{2}$ RA: $\pi/4$ Quad: 2	c. $\cot(330^{\circ}) = -\sqrt{3}$ RA: 30° Quad: 4	d. $\tan(\frac{-\pi}{4}) = -1$ RA: $\pi/4$ Quad: 4
e. $\sec(300^{\circ}) = 2$ RA: 60° Quad: 4	f. $\csc(\frac{5\pi}{6}) = -\frac{2\sqrt{3}}{3}$ RA: $\pi/6$ Quad: 2	g. $\sin(-150^{\circ}) = -\frac{1}{2}$ RA: 30° Quad: 3	h. $\cot(\frac{5\pi}{4}) = 1$ RA: $\pi/4$ Quad: 3
i. $\cos(60^{\circ}) = \frac{1}{2}$ RA: 60° Quad: 1	j. $\sin(\frac{\pi}{6}) = \frac{1}{2}$ RA: $\pi/6$ Quad: 1	k. $\tan(-240^{\circ}) = \sqrt{3}$ RA: 60° Quad: 2	l. $\cos(\frac{4\pi}{3}) = -\frac{1}{2}$ RA: $\pi/3$ Quad: 3

~~* See work @ end of document~~

1. Sum/Difference and Double/Half-Angle Identities

a. Evaluate $\sin 15^\circ$

b. Evaluate $\cos \frac{7\pi}{12}$

c. Evaluate $\cos 20^\circ \cos 25^\circ - \sin 20^\circ \sin 25^\circ =$

d. Evaluate $\frac{\tan 110^\circ + \tan 70^\circ}{1 - \tan 110^\circ \tan 70^\circ} = 0$

e. Given α and β are both angles in quadrant I, $\sin \alpha = \frac{4}{5}$, and $\cos \beta = \frac{5}{13}$, find:

• $\sin(\alpha + \beta) = \frac{56}{65}$

• $\tan(\alpha - \beta) = \frac{-16}{63}$

• $\cos(\alpha - \beta) = \frac{63}{65}$

• $\sin(2\alpha) = \frac{24}{25}$

• $\cos(2\beta) = \frac{-119}{169}$

f. Given $\sin \theta = \frac{12}{13}$, $0 < \theta < \frac{\pi}{2}$, find:

• $\sin(2\theta) = \frac{120}{169}$

• $\cos(2\theta) = \frac{-119}{169}$

• $\sin\left(\frac{\theta}{2}\right)$

g. Given $\tan \theta = \frac{15}{8}$, $\pi < \theta < \frac{3\pi}{2}$, find

• $\sin(2\theta) = \frac{240}{289}$

• $\cos(2\theta) = \frac{-161}{17}$

• $\sin\left(\frac{\theta}{2}\right)$

3. Simplifying expressions

a. Simplify: $\cos \theta \tan \theta \csc \theta = 1$

b. Simplify: $(1 - \sin x)(1 + \sin x) = \cos^2 x$

c. Simplify: $\frac{\csc \theta}{1 + \cot^2 \theta} = \sin \theta$

d. Simplify: $\cos \theta \tan \theta \csc \theta = 0$ mit

e. Simplify: $\frac{1}{\sin^2 \theta} - \frac{1}{\tan^2 \theta} = 1$

f. Simplify: $\sin^2 x + \sin^2 x \cot^2 x = 1$

4. Proving Trig Identities

~~* See all these worked out at end of document.~~

a. Prove: $\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$

b. Prove: $\tan x \sin x + \cos x = \sec x$

c. Prove: $\tan^2 x = \csc^2 x \tan^2 x - 1$

d. Prove: $\cos^2 x = \frac{\csc x \cos x}{\tan x + \cot x}$

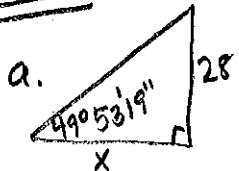
e. Prove: $\frac{\csc \theta}{\sin \theta} - \frac{\cot \theta}{\tan \theta} = 1$

f. Prove: $\cot x + \tan x = \frac{\sec^2 x}{\tan x}$

g. Prove: $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2 \sec x$

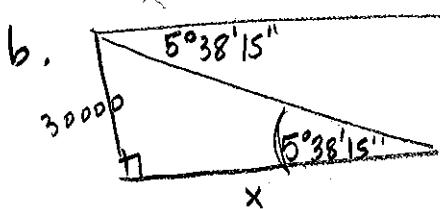
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Part 4



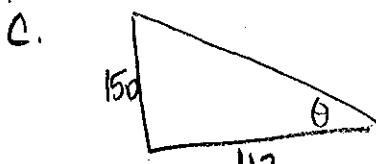
$$\tan 49^\circ 53' 19'' = \frac{28}{x}$$

$$x = 23,588 \text{ ft}$$



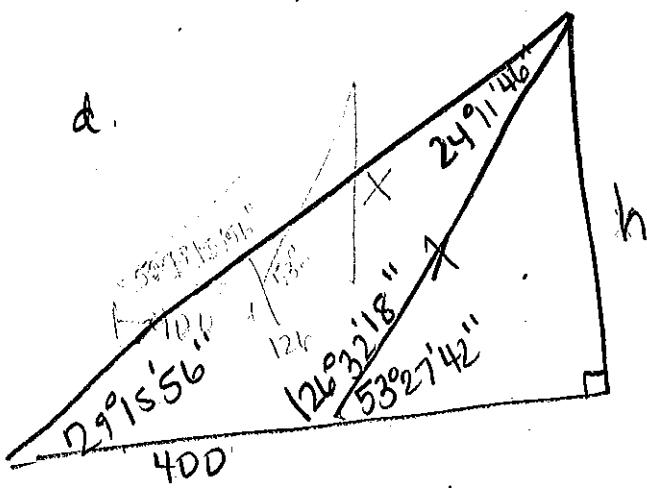
$$\tan 5^\circ 38' 15'' = \frac{30000}{x}$$

$$x = 303915.37 \text{ ft}$$



$$\tan \theta = \frac{150}{42}$$

$$\theta = 74.36^\circ = 74^\circ 21' 27.913''$$



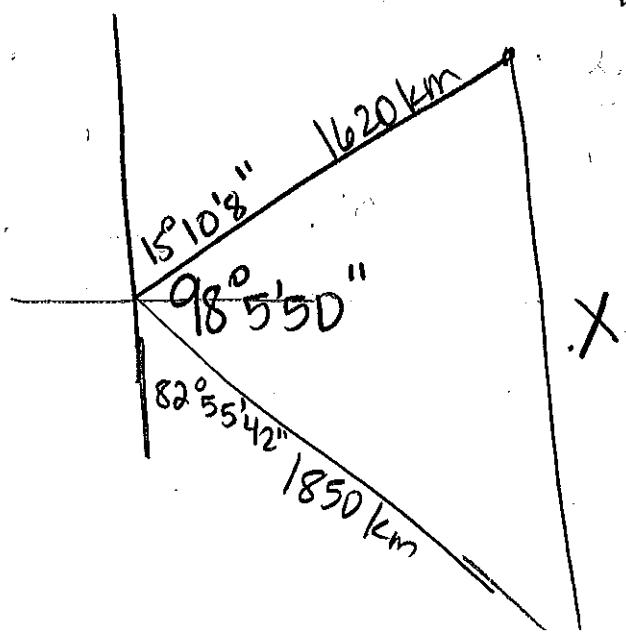
$$\frac{\sin 24^\circ 11' 46''}{400} = \frac{\sin 29^\circ 15' 56''}{x}$$

$$x = 477.096 \text{ ft}$$

$$\sin 53^\circ 27' 42'' = \frac{h}{477.096}$$

$$h = 383.327 \text{ ft}$$

e.



Law of Cosines!

$$x^2 = 1620^2 + 1850^2 - 2(1620)(1850)\cos 98^\circ 5' 50''$$

$$x = 2625.105 \text{ km}$$

Part 7

1) a) $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

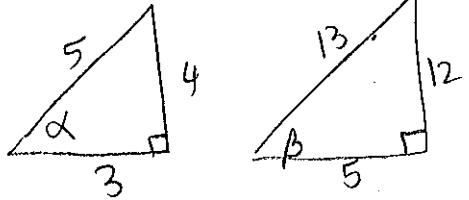
b) $\cos \frac{7\pi}{12} = \cos(\frac{\pi}{4} + \frac{\pi}{3}) = \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}$

$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

c) $\cos 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$

d) $\tan(110^\circ + 70^\circ) = \tan 180^\circ = \boxed{0}$

e) 

• $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$= \frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \boxed{\frac{56}{65}}$$

• $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

$$= \frac{\frac{4}{3} - \frac{12}{5}}{1 + \frac{4}{3} \left(\frac{12}{5}\right)}$$

$$= \boxed{-\frac{16}{63}}$$

• $\sin(2\alpha) = 2 \sin \alpha \cos \alpha$

$$= 2 \left(\frac{4}{5}\right) \left(\frac{3}{5}\right)$$

$$= \boxed{\frac{24}{25}}$$

• $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$$= \frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65}$$

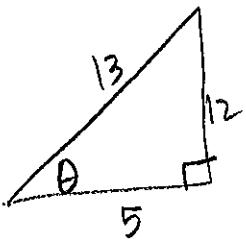
$$= \boxed{\frac{63}{65}}$$

• $\cos(2\beta) = \cos^2 \beta - \sin^2 \beta$

$$= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$= \boxed{-\frac{119}{169}}$$

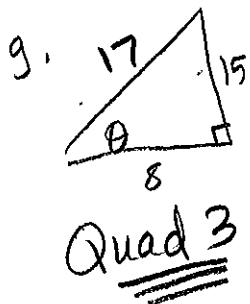
f.)



$$\begin{aligned}\bullet \sin(2\theta) &= 2\sin\theta\cos\theta \\ &= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) \\ &= \boxed{\frac{120}{169}}\end{aligned}$$

$$\bullet \sin\frac{\theta}{2} = \sqrt{\quad}$$

$$\begin{aligned}\bullet \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 \\ &= \boxed{-\frac{119}{169}}\end{aligned}$$



$$\begin{aligned}\bullet \sin(2\theta) &= 2\sin\theta\cos\theta \\ &= 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right) \\ &= \boxed{\frac{240}{289}}\end{aligned}$$

$$\bullet \sin\frac{\theta}{2} = \sqrt{\quad}$$

$$\begin{aligned}\bullet \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= \left(\frac{8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 \\ &= \boxed{-\frac{161}{289}}\end{aligned}$$

Part 7

③ a) $\cos\theta + \tan\theta \csc\theta$

$$\begin{aligned} & \cancel{\cos\theta} \cdot \frac{\sin\theta}{\cancel{\cos\theta}} + \frac{1}{\sin\theta} \\ & = \boxed{1} \end{aligned}$$

b) $(1-\sin x)(1+\sin x)$ ~~FOL~~

$$\begin{aligned} & 1 + \sin x - \sin x - \sin^2 x \\ & 1 - \sin^2 x \\ & \boxed{\cos^2 x} \end{aligned}$$

c) $\frac{\csc\theta}{1 + \cot^2\theta} = \frac{\csc\theta}{\csc^2\theta}$

$$\begin{aligned} & = \frac{1}{\csc\theta} \\ & = \boxed{\sin\theta} \end{aligned}$$

d) $\cos\theta + \tan\theta \csc\theta$
(see a!)

e) $\frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta}$

$$\begin{aligned} & \csc^2\theta - \cot^2\theta \\ & \boxed{1} \end{aligned}$$

f) $\sin^2 x + \sin^2 x \cot^2 x$
 $\sin^2 x (1 + \cot^2 x)$
 $\sin^2 x (\csc^2 x)$
 $\sin^2 x \cdot \frac{1}{\sin^2 x}$
 $= \boxed{1}$

Part 7

④ a)
$$\frac{(\tan\theta + \cot\theta)}{\sin\theta \cos\theta} = \frac{1}{\sin\theta \cos\theta}$$

$$= \frac{\sin\theta \cdot \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \cdot \cos\theta}{\cos\theta \cdot \frac{1}{\sin\theta} + \sin\theta \cdot \cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \boxed{\frac{1}{\cos\theta \sin\theta}} \quad \checkmark$$

d)
$$\frac{\csc x \cos x}{\tan x + \cot x}$$

$$= \frac{1}{\sin x} \cdot \frac{\cos x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

$$= \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \frac{1}{\cos x \sin x}$$

b)
$$\tan x \sin x + \cos x$$

$$= \frac{\sin x}{\cos x} \cdot \sin x + \cos x$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \boxed{\sec x} \quad \checkmark$$

$$= \frac{\cos x}{\sin x} \cdot \frac{\cos x \sin x}{1}$$

$$= \boxed{\cos^2 x} \quad \checkmark$$

c)
$$\csc^2 x \tan^2 x - 1$$

$$= \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} - 1$$

$$= \sec^2 x - 1$$

$$= \boxed{\tan^2 x} \quad \checkmark$$

e)
$$\frac{\csc\theta}{\sin\theta} - \frac{\cot\theta}{\tan\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1 - \cos^2\theta}{\sin^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta}$$

$$= \boxed{1} \quad \checkmark$$

*) This will not be on your test.

$\star f) \cot x + \tan x \rightarrow \star g)$

$$= \frac{\cos x \cdot \csc x}{\sin x \cdot \csc x} + \frac{\sin x \cdot \csc x}{\cos x \cdot \csc x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{1 \cdot \sec^2 x \cdot \sec^2 x}{\sin x \cos x \cdot \sec^2 x}$$

$$= \frac{\sec^2 x}{\sin x \cos x \cdot \frac{1}{\cos^2 x}}$$

$$= \frac{\sec^2 x}{\frac{\sin x}{\cos x}}$$

$$\therefore \frac{\sec^2 x}{\tan x}$$

$$\checkmark$$

$$\begin{aligned} & \frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \frac{(1-\sin x)}{(1-\sin x)} \\ &= \frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{1-\sin x - \sin^2 x} \\ &= \frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{1-\sin^2 x} \\ &= \frac{1+\sin x}{\cos x \cdot \cos x} + \frac{\cos x(1-\sin x)}{\cos^2 x} \\ &= \frac{\cos x(1+\sin x)}{\cos^2 x} + \frac{\cos x(1-\sin x)}{\cos^2 x} \\ &= \frac{\cos x(1+\sin x + 1-\sin x)}{\cos^2 x} \\ &= \frac{2\cos x}{\cos x} \\ &= 2 \sec x \quad \checkmark \end{aligned}$$

*) This will not be on your test.

$\star f) \cot x + \tan x \rightarrow \star g)$

$$= \frac{\cos x \cdot \cot}{\sin x \cdot \cot} \cdot \frac{\sin x^{\sin x}}{\cos x^{\sin x}}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{1 \cdot \sec^2 x \cdot \sec^2 x}{\sin x \cos x \cdot \sec^2 x}$$

$$= \frac{\sec^2 x}{\sin x \cos x \cdot \frac{1}{\cos^2 x}}$$

$$= \frac{\sec^2 x}{\frac{\sin x}{\cos x}}$$

$$= \frac{\sec^2 x}{\tan x}$$

$$\checkmark$$

$$\begin{aligned} & \frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \frac{(1-\sin x)}{(1-\sin x)} \\ & = \frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{1-\sin x + \sin x - \sin^2 x} \\ & = \frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{1-\sin^2 x} \\ & = \frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{\cos^2 x} \\ & = \frac{\cos x(1+\sin x) + \cos x(1-\sin x)}{\cos^2 x} \\ & = \frac{\cos x(1+\sin x + 1-\sin x)}{\cos^2 x} \\ & = \frac{2}{\cos x} \\ & = 2 \sec x \quad \checkmark \end{aligned}$$